WAVE PROPAGATION IN THREE-PHASE MIXTURES OF A GAS WITH PARTICLES AND LIQUID DROPS

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Propagation of weak disturbances and shock wave structure are studied in mixtures of a gas with large liquid drops and small solid particles, in the presence of particle precipitation on the drops. The dependence of wave number on disturbance frequency is obtained. The effect of the defining parameters on the attenuation coefficient and phase velocity of sound is found, together with their effect on flow in the shock wave relaxation zone. Some theoretical and experimental studies of wave propagation in two-phase media can be found in [1-11]. Shock wave structure in gas mixtures with solid particles was studied in [1]. In [2] the effect of phase transitions (evaporation, condensation) on flow in the compression wave relaxation zone was studied in a gaseous suspension of liquid drops. Shock wave structure was studied in the presence of droplet breakup in [3]. In [4] flow in the relaxation zone of intense shock waves was studied with consideration of the effect of particle fusion. A detailed review of shock wave propagation studies in gas suspensions can be found in [5]. In [6] propagation of weak disturbances in a mixture of gas with solid inert particles was studied, while [7, 8] examined mixtures of gas and vapor with liquid drops in the presence of phase transitions. The effect of transient components of interphase interaction upon high frequency disturbances was considered in [9].

1. Basic Assumptions and Phase Equations of State. We will make the assumptions usual in mechanics of multiphase media [12]. In addition we assume that the drops are incompressible, of one size, do not collide or break up, and that the effects of viscosity and thermal conductivity are significant only in the processes of gas phase interaction with the solid and liquid phases, that large drops interact with small particles, and that particles collide with and are captured by drops.

The dimensions of the solid particles are so small that their mixture with the gas can be treated as a single-velocity, single-temperature continuous medium with its own unique thermophysical properties. This medium will be termed an effective gas.

Within the framework of these assumptions the equations of motion of the gas suspension under consideration can be written easily (they can be obtained, for example, from equations in [3, 12]). Here we will present only the equation of state and the phase interaction laws, which indicate certain unique features of gas flow in the presence of particle precipitation on drops. The gas will be considered calorically perfect, while the solid and liquid phases are incompressible media with constant specific heats. Then the equation of state of the effective gas and the large drops can be written in the form

$$
\begin{gathered}
p=\rho_1^0 R_1 T_1,\ e_1=c_1T_1,\ \ e_{1p}=c_pT_1,\ e_2=c_2T_2,\ R_1=x_{1g}R_g,\ c_1=c_{gV}\,x_{1g}+\\
+c_px_{1p},\ \ c_2=c_lx_{2l}+c_px_{2p},\ \ x_{1g}+x_{1p}=1,\ \ x_{2l}+x_{2p}=1;\ \ \rho_p^0,\ \rho_l^0,\ R_g,\\
c_{gV},\ c_p,\ c_l=\text{const},\ x_{1g}=\rho_{1g}/\rho_1,\ x_{1p}=\rho_{1p}/\rho_1,\ x_{2l}=\rho_{2l}/\rho_2,\ x_{2p}=\rho_{2p}/\rho_2,\\
\rho_1=\rho_{1g}+\rho_{1p},\ \ \rho_2=\rho_{2l}+\rho_{2p},\ \ \alpha_1+\alpha_2=1,\ \ \rho_1=\alpha_1\rho_1^0,\ \alpha_1=\alpha_{1g}+\\
+\alpha_{1p},\ \ \alpha_2=\alpha_{2l}+\alpha_{2p},\ \ \alpha_{2l}=\rho_{2l}/\rho_l^0,\ \alpha_{2p}=\rho_{2p}/\rho_p^0,\ \alpha_{1p}=\rho_{1p}/\rho_p^0.
\end{gathered}
$$

Here p, e_1, e_2, T_1, T_2 are the gas pressure, and internal energies and temperatures of the effective gas and large drops; e_{1p} is the internal energy of the small particles; ρ_1, ρ_2, ρ_{1g}, ρ_{1p}, $\rho_{2\ell}$, ρ_{2p} are the reduced densities of the effective gas, large drops, and their components: $\rho_1{}^0$, $\rho_p{}^0$, $\rho_\ell{}^0$ are the true densities of the effective gas, fine particles, and liquid component of the large drops; α_{1g}, α_{1p}, $\alpha_{2\ell}$, α_{2p} is the volume content of the mixture components; R_g, c_{gV}, c_p, c_ℓ are the ideal gas content, specific heat of the gas (at constant volume), the small particles, and the liquid component of the large drops.

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2. Phase Interaction Laws. To define the intensity of fine particle capture by large drops we will use an elementary scheme of counting collisions between particles and drops. We will consider an isolated drop of diameter d, moving at velocity $\mathbf{v}_2$ in an equilibrium mixture of gas with small particles having a velocity $\mathbf{v}_1$. Over a unit time there will collide with this drop all particles located at the given moment within the volume $\pi d^2|\mathbf{v}_1 - \mathbf{v}_2|/4$. However, under the influence of the gas flow some particles arriving at the drop may change their trajectories and not collide, while some that do collide may undergo specular reflection from the surface. Considering these facts, we write the expression for intensity of capture of fine particles in the form

$$j = \eta\varkappa(\pi d^2/4)\rho_{1p}|\mathbf{v}_1 - \mathbf{v}_2|, \ \eta = \eta(d_p, \rho_p^0, \mu_g, d, \ldots). \tag{2.1}$$

Here η is a correction coefficient, characterizing the efficiency of particle collision with the drop, and dependent on the conditions of the flow of the mixture of particles and gas over the drop (usually the coefficient η can be expressed as a function of $\sqrt{\mathrm{Stk}}$ or $\sqrt{\psi} = \sqrt{L_{pv}/d}$ [13], where $\mathrm{Stk} = 2L_{pv}/d$ is the Stokes number and L_{pv} is the characteristic length of change in particle velocity upon approach to the drop); d_p is the diameter of the small particles; μ_g is the dynamic viscosity of the gas; κ is the precipitation coefficient, indicating the fraction of particles colliding with the drop which are captured by it (the remaining fraction $1 - \kappa$ is specularly reflected from the drop surface).

We specify the interaction between the drop and the carrier medium in the following manner:

$$f = (\pi d^2/8)\rho_{1g}^0 C_d|\mathbf{v}_1 - \mathbf{v}_2|(\mathbf{v}_1 - \mathbf{v}_2) \tag{2.2}$$

(C_d is the drop resistance coefficient). Over a wide range of change of the defining parameters we may write C_d as [4, 14]

$$C_d = \begin{cases} 27\mathrm{Re}_{12}^{-0{,}84}, & \mathrm{Re}_{12} < 80, \\ 0{,}27\mathrm{Re}_{12}^{0{,}217}, & 80 \leqslant \mathrm{Re}_{12} < 10^4, \\ 2, & 10^4 \leqslant \mathrm{Re}_{12}, \ \mathrm{Re}_{12} = \rho_{1g}^0 d|\mathbf{v}_1 - \mathbf{v}_2|/\mu_g \end{cases} \tag{2.3}$$

(Re_{12} is the Reynolds number relative to flow over the drop).

The intensity of heat input to the drop from the carrier phase can be written as [2, 5]

$$q = \pi d\lambda_g \mathrm{Nu}_1(T_1 - T_2), \ \mathrm{Nu}_1 = 2 + 0{,}6\mathrm{Re}_{12}^{0{,}5}\mathrm{Pr}_1^{0{,}3}, \ \mathrm{Pr}_1 = \frac{c_{gp}\mu_g}{\lambda_g}. \tag{2.4}$$

Here Nu_1 and Pr_1 are Nusselt and Prandtl numbers; c_{gp}, λ_g are the specific heat of the gas at constant pressure and its thermal conductivity coefficient. At low Reynolds numbers ($\mathrm{Re}_{12} \ll 1$) for C_d and Nu_1 we may use simpler expressions [5, 12]: $C_d \cong 24/\mathrm{Re}_{12}$, $\mathrm{Nu}_1 \cong 2$.

It should be noted that the inertial effect is significant in particle collisions with the drop if $\mathrm{Stk} = 2L_{pv}/d$ takes on sufficiently high values ($\mathrm{Stk} \gg 1$). For low Stokes numbers ($\mathrm{Stk} \ll 1$) precipitation of fine particles occurs basically due to their diffusion to the drop surface. The contribution of the inertial effect to the particle capture by drops is quite small in this case ($\eta \cong 0$) [13], since upon arrival near the drop the small particles can change their trajectories and flow around it, following flow lines. Considering this, as well as the fact that some particles are reflected from the surface upon collision with the drop, we write the expression for precipitation of particles on a single drop in the diffusion regime in the form $j = \beta\kappa\pi d^2 m_p(n_{p\Sigma} - n_p)$. Here $n_{p\Sigma}$, n_p are the concentrations of particles on the drop surface and far therefrom; m_p is the mass of one small particle; β is the diffusion velocity dependent on the properties of the small particles, the drops, and the carrier medium, and determined experimentally or from some other considerations; the coefficient κ, as in Eq. (2.1), indicates the fraction of particles colliding with the drops which precipitate. We will assume below that $\kappa = 1$ and $n_{p\Sigma} = 0$ (i.e., all particles falling on the drop surface precipitate into it).

We will note that the diffusion velocity β can be represented as the ratio of the particle diffusion coefficient to the characteristic thickness of the diffusion (concentration) layer at the drop surface [15]. At present reliable experimental and theoretical data on the particle diffusion coefficient at the drop surface are lacking.

3. Dispersion Relationships. We will consider propagation of planar periodic waves in a gas suspension of fine particles and large liquid drops (gas–dust–drop mixture). We will assume that the undisturbed gas–dust–drop mixture is in thermodynamic equilibrium ($v_{10} = v_{20}$, $T_{10} = T_{20}$) and precipitation of the fine particles on drops does not occur.

To study propagation of weak disturbances in such a medium (where $Re_{12} \ll 1$, $Stk \ll 1$) we linearize the equations of motion and seek a solution of the linear system thus obtained in the form of a decaying traveling wave $\exp[i(kx - \omega t)]$. The condition for existence of a nontrivial solution of that type leads to the following relationship between the wave vector k and the dimensionless frequency of the disturbance $\sigma = \omega\tau_T/a_g$:

$$\begin{gathered}
k^2 = \sigma^2 \frac{\alpha_{1g0}\gamma_g}{r_g\tau_T^2} \frac{(s - i\sigma\tau_m/\tau_T)(\Pi_1 - \bar{\alpha}_{10} i\sigma\tau_v/\tau_T)(C_V - i\sigma C_{1V})}{(C_V - i\sigma C_{1V})(\Pi_2 + \Pi_3) + \Pi_4/\gamma_g},\\
\Pi_1 = \bar{\alpha}_{10} + \alpha_{20},\ \Pi_2 = \alpha_{20}(1 - \bar{\alpha}_{10} i\sigma\tau_v/\tau_T)[s + (1 - 1/r_p) - i\sigma\tau_m/\tau_T],\\
\Pi_3 = (1 - \alpha_{10} i\sigma\tau_v/\tau_T)[s(\alpha_{1g0} + r_p\alpha_{1p0}) - \alpha_{10} i\sigma\tau_m/\tau_T],\\
\Pi_4 = (1 - i\sigma)(s - i\sigma\tau_m/\tau_T)\left[1 - (\alpha_{10}^2 + \bar{\alpha}_{10}\alpha_{20})\, i\sigma\tau_v/\tau_T\right],\\
C_V = C_{gV} + m_{1p}C_p + m_2C_l,\ C_{1V} = C_{gV} + m_{1p}C_p,\\
m_{1p} = \rho_{1p0}/\rho_{1g0},\ m_2 = \rho_{20}/\rho_{1g0},\ s = \alpha_{20}/r_p\alpha_{1p0},\\
\tau_m = a_g t_m,\ \tau_v = a_g t_v,\ \tau_T = a_g t_T,\\
t_m = \frac{0{,}17d}{\beta r_p\alpha_{1p0}},\ t_v = \frac{\rho_l^0 d^2}{18\mu_g},\ t_T = \frac{\rho_l^0 d^2 c_l}{12\lambda_g},\\
r_g = \rho_{1g0}^0/\rho_l^0,\ r_p = \rho_p^0/\rho_l^0,\ \bar{\alpha}_{10} = r_g\alpha_{1g0} + r_p\alpha_{1p0},\\
C_{gV} = c_{gV}/\gamma_g R_g,\ C_p = c_p/\gamma_g R_g,\ C_l = c_l/\gamma_g R_g.
\end{gathered} \tag{3.1}$$

Here t_m, t_v, and t_T are the characteristic times for change in mass of an individual drop due to precipitation of small particles thereon, and for relaxation of the drop velocity and temperature in the Stokes flow regime; τ_m, τ_v, τ_T are the reduced relaxation times, having dimensions of length; m_{1p}, m_2 are the relative mass contents of fine particles and large drops; γ_g and a_g are the adiabatic index of the gas and the speed of sound therein.

In the limiting cases we can obtain from Eq. (3.1) expressions for the equilibrium $a^e(\sigma \to 0)$ and frozen $a^f(\sigma \to \infty)$ speed of sound in the gas–drop mixture, in the presence of dust (fine particle) precipitation on the drops:

$$\begin{gathered}
a^e = a_g\left[\frac{\Gamma - 2\alpha_{1p0}(1 - r_p)}{\gamma_g\alpha_{1g0}(1 + m_{1p} + m_2)}\right]^{1/2},\\
a^f = a_g\left[\frac{r_g^2}{\gamma_g}\frac{m_*}{1 + m_{1p}}\Gamma_1\right]^{1/2},\\
\Gamma = \frac{C_{gP} + m_{1p}C_p + m_2C_l}{C_{gV} + m_{1p}C_p + m_2C_l},\ \Gamma_1 = \frac{C_{gP} + m_{1p}C_p}{C_{gV} + m_{1p}C_p},\\
m_* = (1/r_g + m_{1p}/r_p)^2 + m_2(1 + m_{1p}).
\end{gathered} \tag{3.2}$$

In the absence of particle precipitation on drops ($j \equiv 0$) the dispersion relationship has the form

$$\begin{gathered}
k^2 = \sigma^2 \frac{\alpha_{1g0}}{r_g\tau_T^2} \frac{(\Pi_1 - \bar{\alpha}_{10} i\sigma\tau_v/\tau_T)(\gamma_g C_P/C_{1P*} - i\sigma)(C_{1P*}/C_{1P})}{\left[1 - (\alpha_{10}^2 + \bar{\alpha}_{10}\alpha_{20})\, i\sigma\tau_v/\tau_T\right](\Gamma C_P/C_{1P} - i\sigma)},\\
C_P = C_{gP} + m_{1p}C_p + m_2C_l,\ C_{1P} = C_{gP} + m_{1p}C_p,\ C_{1P*} = \gamma_g C_{1V}.
\end{gathered}$$

In this case the expression for the equilibrium and frozen speeds of sound will be:

$$a_0^e = a_g\left[\frac{\Gamma}{\gamma_g\alpha_{1g0}^2(1 + m_{1p} + m_2)}\right]^{1/2},\ a_0^f = a^f. \tag{3.3}$$

Comparing Eqs. (3.2) and (3.3), we note that at $\alpha_{1p0} \ll 1$ the presence of particle precipitation on the drops has an insignificant effect on the equilibrium speed of sound a^e, and it may lead to either an increase (for $r_p > 1$) in the equilibrium velocity, or a decrease (at $r_p < 1$). The presence of particle precipitation has no effect on the frozen speed of sound a^f. It is interesting that at $r_p = \rho_p^0/\rho_\ell^0 = 1$ the process of particle precipitation has no effect on the dependence of wave vector on frequency of the external disturbance.

4. Formulation of the Problem and Conditions for Similarity of Shock Wave Structures. Let a planar shock wave propagate at velocity v_{10}, where $v_{10} > a^e, a^f$, in an infinite space filled by a mixture of gas with fine particles and coarse drops. Then there will be a discontinuity in the effective gas ahead of the shock wave, upon which the effective gas parameters satisfy the Rankin–Hugoniot relationships, and the parameters of the coarse drops undergo practically no change. Nonequilibrium of velocity and temperature behind the discontinuity lead to formation of a relaxation zone where mass, momentum, and heat exchange occurs between the drops and effective gas.

The parameters of the mixture components behind the discontinuity define boundary conditions at some point $x = x_f$, corresponding to the position of the compression discontinuity, and permit calculation of the relaxation zone structure in the region $x > x_f$.

Now let $a^e < v_{10} < a^f$. Then there is no discontinuity before the shock wave, i.e., the mixture parameters in the comparison wave change continuously from the equilibrium state ahead of the wave to the equilibrium state behind it [2, 5]. In this case to formulate boundary conditions we can use the solution of the linearized system of equations of mixture motion in the vicinity of the initial state ahead of the wave. The formulation of boundary conditions for calculation of shock wave structure in gas–drop mixtures was considered in greater detail in [2, 3].

We will estimate the characteristic times for change in drop velocity, temperature, and mass behind the shock wave front. At high Reynolds numbers ($Re_{12} \gg 1$), where Newtonian conditions are realized for flow of the gas over the drop, the characteristic velocity t_v^N and temperature t_T^N relaxation times have the form [16]

$$t_v^N \cong \frac{2{,}6\rho_l^0 d}{\rho_{1g0}^0 v_{10}}, \quad t_T^N \simeq \frac{\rho_l^0 d^2 c_l}{5{,}1\lambda_g (\mathrm{Re}_{0s}\mathrm{M})^{0,5}\,\mathrm{Pr}_1^{0,33}},$$
$$\mathrm{Re}_{0s} = \rho_{1g0}^0\, d a_{g0}/\mu_g, \quad \mathrm{M} = v_{10}/a_{g0}.$$

Here Re_{0s}, M are the characteristic Reynolds and Mach numbers. Estimates show that $t_T^N/t_v^N \gg 1$, i.e., for $Re_{12} \gg 1$ the characteristic time for equalization of gas and drop temperature significantly exceeds the time for velocity equalization.

In the case where the drop moves in an equilibrium mixture of gas with fine particles, the characteristic time for change in drop mass (due to inertial capture of particles) $t_m^N \cong 0.2\rho_\ell{}^0 d/\eta\rho_{1p0}v_{10}$. It follows from the expressions for t_v^N and t_m^N that $t_m^N/t_v^N \cong 0.08\rho_{1g0}{}^0/\eta\rho_{1p0}$. It is evident that at $\rho_{1g0}{}^0 \sim \rho_{1p0}$ and $\eta \sim 1$ as regards t_m^N and t_v^N we have $t_m^N \ll t_v^N$, i.e., the characteristic time for increase in drop mass due to capture of fine particles is much less than the characteristic time of drop velocity relaxation. It should be noted that the expression for t_m^N was obtained without consideration of the effect of change in drop velocity upon particle capture (i.e., it was obtained assuming constancy of drop motion velocity in the effective gas), while the expression for t_v^N is for the case where fine particles are absent from the gas flow. In connection with this the t_m^N and t_v^N values are basically methodical in character.

To analyze the conditions of shock wave structure similarity in the mixture of gas with fine particles and large drops we will analyze three cases.

A. The mixture consists of a gas with drops (no fine particles). The similarity criteria then consist of seven dimensionless parameters: γ_g, M, $m_2 = \rho_{20}/\rho_{10}$, α_{1g0}, $C = c_\rho/c_{gV}$, Re_{0s}, Pr_1. The dominant effect on wave structure in gas suspensions is usually interphase friction, so that we can consider γ_g, M, and m_2 to be the basic criteria for approximate flow similarity in the relaxation zone.

B. The mixture consists of a gas with fine particles and large drops, but interaction between particles and drops is absent ($j = 0$). Then among the parameters for approximate similarity of shock wave flow structure we have the relative mass content of fine particles ahead of the wave $m_{1p} = \rho_{1p0}/\rho_{1g0}$.

C. Interaction between the particles and drops does exist in the mixture of gas with fine particles and large drops ($j \neq 0$). In this case the basic criteria for approximate similarity of shock wave structure will be the dimensionless quantities γ_g, M, M_{1p}, m_2, η.

5. Analysis of Calculation Results. The effects of precipitation of fine particles on large drops as well as that of the basic parameters on the character of dispersion relationships and shock wave structure were studied. We considered propagation of weak disturbances in mixtures of air with fine particles of charcoal and large water drops under normal conditions (p_0 = 0.1 MPa, T_{10} = 293 K). The controlling parameters (mass contents of drops $m_2 = \rho_{20}/\rho_{1g0}$ and fine particles $m_{1p} = \rho_{1p0}/\rho_{1g0}$, drop diameter d) were varied over the ranges m_2 = 0.5-2, m_{1p} = 0-2, d = 10-50 μm.

Figures 1, 2 show curves reflecting the character of the dependence of phase velocity $U_p = v_p/a_g$ and attenuation coefficient δ on dimensionless frequency of the external disturbance $\sigma = \omega\tau_T/a_g$. Calculations showed that over a wide range of change in particle diffusion velocity $10^{-4} < \beta < 10$ m/sec the process of particle precipitation on drops has practically no effect on the dependence of U_p and δ on frequency σ. In connection with this, one and the same curve on the figures corresponds to both presence and absence of precipitation. Figure 1 illustrates the effect of drop diameter on the dependence of attenuation frequency and speed of sound on external disturbance frequency in the presence ($m_{1p} \neq 0$) and absence ($m_{1p} = 0$) of fine particles in the effective gas composition. Curves 1-3 correspond to d = 10, 20, and 30 μm for one and the same mass content (m_2 = 1), with the dashed and solid lines being fine particle mass contents of m_{1p} = 0 and m_{1p} = 1.

It is evident that change in drop size strongly affects the disturbance attenuation coefficient, a decrease in drop diameter leads to more rapid attenuation of sound. The most significant process affecting the dispersion relationships is usually interphase friction. With decrease in drop size the intensity of interphase friction increases, which leads to an increase in viscous dissipation of the disturbance energy. It is interesting that change in drop diameter has practically no effect on the dependence of phase velocity U_p on dimensionless disturbance frequency σ.

The effect of mass content of fine particles m_{1p} on attenuation coefficient and phase velocity of disturbances is shown in Fig. 2. Curves 1-3 correspond to contents m_{1p} = 0.5, 1, and 2, while the dashed lines are for absence of fine particles in the mixture (m_{1p} = 0). The mass content of large drops and their diameter are fixed: m_2 = 1, d = 20 μm. It is evident that increase in mass content of fine particles in the mixture leads to a marked decrease in the attenuation coefficient and phase velocity of the disturbances.

We will note that in the external disturbance frequency range considered ($0 < \omega t_T \leq 20$) the contribution of transient Basset, Archimedes, and combined mass forces to the net interphase interaction is quite low. However, at very high frequency ($\omega t_T \geq 10^2$) because of the nonsteady nature of flow over the drop by the gas these interphase forces are comparable to the quasisteady interphase Stokes friction force, Eq. (2.2), or even exceed the latter. Therefore, consideration of nonsteady effects basically affects the disturbance attenuation coefficient, with a quite weak effect on the phase velocity [9, 12].

The structure of a shock wave in a mixture of air with fine graphite dust particles and large water drops was also studied. It was assumed that the mixture was in thermodynamic equilibrium ahead of the wave ($v_{10} = v_{20}$, $T_{10} = T_{20}$) at a pressure of 0.1 MPa. The equations of motion of the gas mixture with Eqs. (2.1)-(2.4) and corresponding boundary conditions were integrated numerically by a modified Euler method. The accuracy of calculations

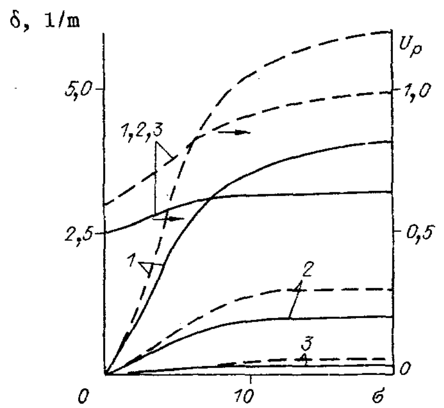

Fig. 1

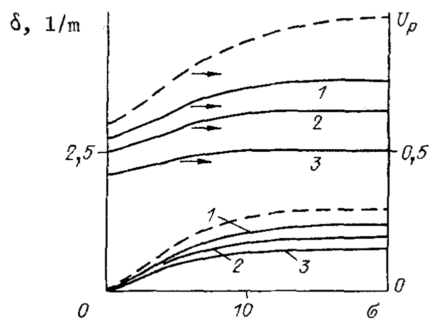

Fig. 2

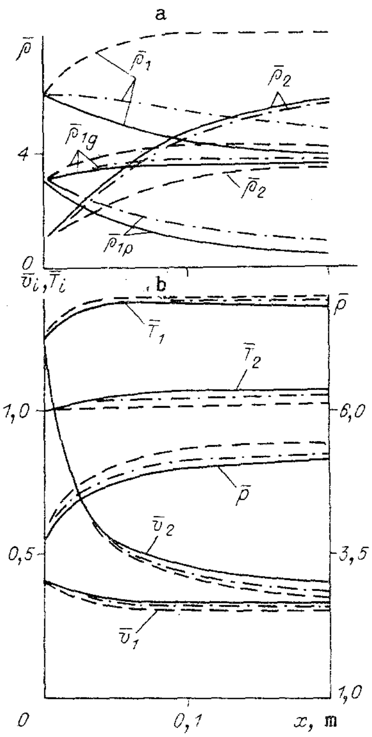

Fig. 3

was monitored by taking the first integrals of mass, momentum, and energy. The calculations were performed for waves with $M = v_{10}/a_{g0} = 0.6$-1.2. The relative mass content of fine particles and large drops was varied from 0.5 to 2. The drop diameter was varied from 50 to 200 μm. The coefficient describing efficiency of particle capture by the drop η was considered constant and varied from 0 to 0.8 (η = 0 corresponds to absence of particle precipitation).

Some results illustrating the effect of particle capture by drops on mixture flow in the relaxation zone of a wave of intensity M = 1.2 at $m_2 = 1$, $m_{1p} = 1$, and d = 200 μm are shown in Fig. 3. The dashed, dash—dot, and continuous curves correspond to η = 0, 0.5, and 0.8. Shown is the behavior of density of the effective gas $\bar{\rho}_1 = \rho_1/\rho_{1g0}$, its components $\bar{\rho}_{1g} = \rho_{1g}/\rho_{1g0}$, $\bar{\rho}_{1p} = \rho_{1p}/\rho_{1g0}$, and the drops $\bar{\rho}_2 = \rho_2/\rho_{1g0}$, together with their velocities $\bar{v}_i = v_i/a_{g0}$, temperatures $\bar{T}_i = T_i/T_{10}$ (i = 1, 2), and gas pressure $\bar{p} = p/p_0$ in the relaxation zone (the additional subscript 0 indicates conditions ahead of the wave).

It is evident that in the presence of particle capture by drops the reduced density of particles $\bar{\rho}_{1p}$ in the relaxation zone decreases, while in the absence of particle precipitation it increases (Fig. 3a). Two significant factors affect the reduced particle density: braking of the effective gas due to the pressure gradient (leading to increase in fine particle concentration) and their precipitation on drops. Depending on which of these dominates, the reduced fine particle density in the wave relaxation zone may either increase or decrease.

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ANALYTICAL AND NUMERICAL INVESTIGATION OF THE THREE-DIMENSIONAL VISCOUS SHOCK-LAYER ON BLUNT SOLIDS

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Three-dimensional problems of viscous flow around bodies are presently among the most pressing problems of hypersonic aerodynamics in connection with the development of craft which move in the upper atmospheric layers. The use of numerical methods in solving such problems requires great amounts of computer time and internal computer memory. Therefore, development of approximate methods which, while being sufficiently accurate, can be used in engineering practice, is very timely. Many approximate methods have been developed for large Reynolds numbers Re. They are based on the boundary layer theory and require knowledge of the parameters of nonviscous flow at the surface of the solid. However, there are presently no similar methods suitable for solving three-dimensional problems of viscous flow around solids at small and medium Reynolds numbers ($Re \leq 10^3$), where the viscosity is considerable throughout the entire region of perturbed flow and the classical boundary layer theory is inapplicable.

On the basis of an approximate solution of the equations of a three-dimensional hypersonic viscous shock-layer, we have obtained an analytical solution for determining the thermal flux and the friction stress at the lateral surface of blunt solids for small and medium Re numbers with an allowance for the slippage effect and the temperature jump at the surface. For a flow characterized by medium or large Re values, a simple expression has been derived for the thermal flux distribution over the surface; the thermal flux is reduced to its value at the stagnation point. This expression depends only on the geometry of the body in the flow. The present article is a continuation of [1], where a similar problem was solved in the neighborhood of the symmetry plane.

1. Consider the steady-state, three-dimensional hypersonic flow of a viscous gas around a smooth, blunt solid at small and medium Re numbers. The flow is investigated by

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